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DEVELOPMENT OF A RAY TRACING COMPUTER MODEL FOR THE PREDICTION OF THE SOUND FIELD IN LONG ENCLOSURES

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A ray tracing computer model, NORMAL, has been developed for predicting the sound field in any shaped enclosed space without curved surfaces, including long enclosures of rectangular cross-section. The model was developed specifically as a basis for the modelling of speech intelligibility in underground stations. The model predicts sound propagation throughout a space, and the impulse response at defined receiver points from which various acoustic parameters for the assessment of speech intelligibility can be calculated. This paper describes the way in which NORMAL models the space, receivers and sources, and the mathematics involved in tracing a ray and calculating the energy contributed to a receiver. The model has been tested in two hypothetical spaces to show that it is capable of predicting the characteristics of the sound fields that would be expected both in a diffuse space, and in a long, non-diffuse, enclosure. The reverberation time tail compensation, used to compensate for the loss of later reflections in the predicted energy decay curve, is described. The effects of including this compensation method on predicted reverberation time in diffuse and non-diffuse spaces are discussed, together with the feasibility of applying this correction method to long enclosures.

1. INTRODUCTION

In recent years, several investigations have been undertaken into the acoustics of long enclosures [1–4]. One of the primary motives for these studies has been the need to understand and predict the sound field in underground stations, in order to improve the speech intelligibility of station announcements [5, 6]. It has been found that the theory of classical room acoustics is not applicable to long enclosures as the sound field is non-diffuse. Much of the investigative work on the acoustics of long enclosures has been carried out using physical scale models [6]. While there are many computer models developed for the prediction of the sound field in spaces such as factories or concert halls, there has been little previous work on the application of these models to long enclosures. The only computer model previously developed specifically for use in long enclosures is that of Kang [7] which is an image source model for use in long enclosures of rectangular cross-section and relatively simple geometry.

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This paper describes the initial stages in the development of ray tracing models for modelling the acoustics of long enclosures, for use ultimately in the prediction of speech intelligibility in underground stations. A ray tracing computer model, NORMAL, was developed to model any shaped space without curved surfaces, including long enclosures of rectangular cross-section. The model predicts sound propagation throughout the space, and also the impulse response from which early decay time, reverberation time and clarity are calculated. Two further models have been developed, based upon the model described here, for the prediction of speech intelligibility in long enclosures of both circular and rectangular cross-section. These models have been validated using data from scale model and real stations [8, 9], and used to investigate the sound field in underground stations and ways in which the speech intelligibility can be improved [10].

The general development of the model NORMAL is described here, together with the validation, in two hypothetical spaces, of its ability to accurately predict the sound field in both diffuse and non-diffuse spaces. The use of the reverberation time tail compensation, to compensate for lack of later reflections in the predicted energy decay curve, is discussed in detail. Predictions of reverberation times using NORMAL with and without the reverberation tail compensation are presented for comparison, and in order to examine the feasibility of using this method in long enclosures.

2. DEVELOPMENT OF THE RAY TRACING COMPUTER MODEL NORMAL

The computer model NORMAL was developed for the prediction of sound propagation, in terms of sound pressure levels, and parameters relating to speech intelligibility including reverberation time, early decay time, and clarity. The latter parameters are derived from the predicted impulse response which is calculated at each receiver position.

The ray tracing method was chosen as it is suitable for the modelling of all cross-sectional shapes including those with curves, and the ultimate aim was to develop a model applicable to long enclosures such as underground stations which may be of rectangular or curved cross-section. NORMAL incorporates those features of ray tracing models which were thought to be most appropriate for prediction in long, disproportionate, enclosures. The number of reflections, or reflection order, is usually used to determine the ending of the tracing of a ray. However, in a long enclosure, the number of reflections depends upon the direction of travel of a ray, and there can be a very great difference in the number of reflections of each ray. For example, rays travelling along the length of the enclosure will have far fewer reflections than rays travelling across the space. For this reason the method of terminating the tracing of a ray is based upon its energy, as described below, instead of the number of reflections. As some rays travel long distances the attenuation by the air is important and therefore air absorption is taken into account in the NORMAL model. The method of determining reflection points was also chosen as being appropriate to spaces of this type.

2.1. MODELLING OF THE SPACE

NORMAL is designed to model convex spaces without curved surfaces, although curved surfaces can be represented approximately by using a series of planes. The surfaces of the space are defined by their corner co-ordinates and plane equations.

The sources in NORMAL can be modelled as omni-directional or directional. An omni-directional source is modelled by a large number of rays emitting from the source uniformly. The directions of the rays emanating from the source can be modelled using random numbers generated by the computer or using the method of Krokstad [11]. The number of rays emitted from the source, N_{ray}, must be large enough to accurately model the uniform distribution of the rays, and is based on the relative volumes of the space and the receivers. The number N_{ray} is calculated from the following formula, as used by Ondet and Barbry [12] in their original ray tracing model for the prediction of sound in industrial workshops:

$$N_{ray} = \frac{10 \times V_{space}}{V_{receiver}}, \tag{1}$$

where V_{space} and $V_{receiver}$ are the volumes of the space and receiver respectively.

The accuracy of predictions using this number of rays has been discussed by Dance and Shield [13].

The sound propagation is simulated by a number of rays travelling within the enclosure, the energy carried by each ray being determined by the number of rays and sound power level of the source. For an omni-directional source with sound power level L_w dB, if the number of the rays emitted from the source is N_{ray}, then the initial energy carried by each ray, E_0, is given by

$$E_0 = \frac{10^{L_w/10}}{N_{ray}} \times 10^{-12}. \tag{2}$$

For a directional source, the energy carried by each ray in a certain direction is decided by the source directivity. If E_0 is the average energy carried by rays according to equation (2), the energy carried by the ray in the θ direction, E_θ, is given by

$$E_\theta = E_0 \times Q_\theta, \tag{3}$$

where Q_θ is the directivity factor of the source.

The number of rays emitted from a directional source is the same as for an omni-directional source.

The receivers are simulated by spheres of radius 0·5 m. In a diffuse sound field, the sound pressure level is uniform throughout, so the positions of individual receivers are relatively unimportant. For a non-diffuse sound field, since the sound field is non-uniform, the positions of receivers are critical. When a ray passes through the receiver's volume, the energy to be added to the receiver is determined using equations (4) and (5) below.

For each receiver point, NORMAL calculates the impulse response curve of $E(t)$ against time t, the sound pressure level, the decay curve, the early decay time, reverberation time, and clarity index.

2.2. TRACING OF A RAY

A ray emitted from an omni-directional source with initial energy E_0, travels in its original direction. After it hits the boundary of the enclosure for the first time, the energy carried by the ray will be reduced to E_l due to the air absorption and surface absorption. The time and distance it travelled from the source until hitting the boundary are t_l and d_l respectively. After n reflections, its energy will be reduced to E_n, the time and distance travelled from the last reflection point being t_n and d_n. Thus, when it arrives at the receiver position after n reflections, if the energy carried is E', and the time and distance travelled from the last reflection are t' and d', then the total time and distance travelled from the source are t and d where

$$t = t_1 + t_2 + \cdots t_n + t',$$

$$d = d_1 + d_2 + \cdots d_n + d'.$$

If the distance travelled by the ray within the receiver volume is d_{cell} and the intensity sensed by the receiver volume is $E(t)$, then

$$E(t) = \frac{E' \times d_{cell}}{V_{receiver}}, \tag{4}$$

where $V_{receiver}$ is the receiver volume, and E' is energy carried by the ray when it hits the receiver given by

$$E' = E_0 e^{-h \cdot d} \prod_i (1 - \alpha_i), \tag{5}$$

when E_0 is the original energy of the ray, h is the air absorption attenuation and α_i is the surface absorption coefficient of the ith plane.

The NORMAL model uses the energy discontinuity percentage (EDP) as defined by Dance [14], rather than the number of reflections, to decide when to terminate the tracing of a ray. The EDP represents the percentage of a ray's energy lost before the tracing of the ray is terminated. Typically, the EDP has a value of between 90 and 99%, the lower the value then the shorter the run time of the model. For diffuse spaces with uniform absorption, the relationship between the reflection order and EDP is as follows:

$$n = \frac{\ln(1 - P/100)}{\ln(1 - \alpha) - hl'} \tag{6}$$

where n is the reflection order; P is the energy discontinuity percentage, h is the air absorption attenuation (dB/m), l is the mean free path length and α is the average absorption coefficient.

The advantage of using EDP is that, in effect, the reflection order is optimized for each individual space being modelled. Furthermore, using the EDP is more representative of the real situation than using reflection order as in the real case the ending of the travel of a ray is determined by its loss of energy and not by the number of reflections. It was therefore decided to use EDP to terminate the tracing of rays in this model.

The method used in NORMAL to determine the locations of reflection points on the enclosure surfaces is that of Kulowski [15].

3. VALIDATION OF NORMAL IN TWO HYPOTHETICAL SPACES

In order to examine the ability of NORMAL to model both diffuse and non-diffuse spaces, the sound propagation and reverberation times in two hypothetical spaces were predicted by NORMAL and compared with values calculated according to classical room acoustics. One space was a quasi-cubic empty space with quasi-uniform distribution of the surface absorption, for which classical theory is applicable; the other was a long rectangular space with reflecting ends, which is not suitable for the application of classical theory. If NORMAL is capable of modelling both diffuse and non-diffuse sound fields the predictions by NORMAL should agree with the theoretical calculations in the quasi-cubic space and disagree with calculated values in the long enclosure. This investigation was therefore used to determine the suitability of NORMAL for use in long enclosures, and as a basis for a model for the prediction of speech intelligibility in underground stations.

3.1. THE QUASI-CUBIC SPACE

The quasi-cubic space was assumed to have dimensions $10 \times 9 \times 8\ m^3$; the surface absorption coefficients for all six surfaces were taken to be 0·1. The source was 1 m away from all three nearest planes surrounding it, the receiver positions were at various points within the room. The receiver volume was a sphere with radius 0·5 m, and the sound power level of the source was assumed to be 90 dB. The co-ordinates of the receiver positions are given in Table 1, and the space is illustrated in Figure 1.

3.1.1. *Comparison between classical theory and model predictions*

According to the Sabine formula, the reverberation times with and without air absorption in this space are 2·38 and 2.39 s respectively.

The sound pressure levels predicted by NORMAL and calculated using Sabine theory at each receiver position, together with the differences between the predicted

TABLE 1

Co-ordinates of receiver positions in quasi-cubic space

Positions	A	B	C	D	E	F	G
x co-ordinate	4	4	2·5	2·5	2·5	7·5	7·5
y co-ordinate	5	9	4	9	6	6	9
z co-ordinate	4·5	4·5	2·5	2·5	2·5	7·5	7·5

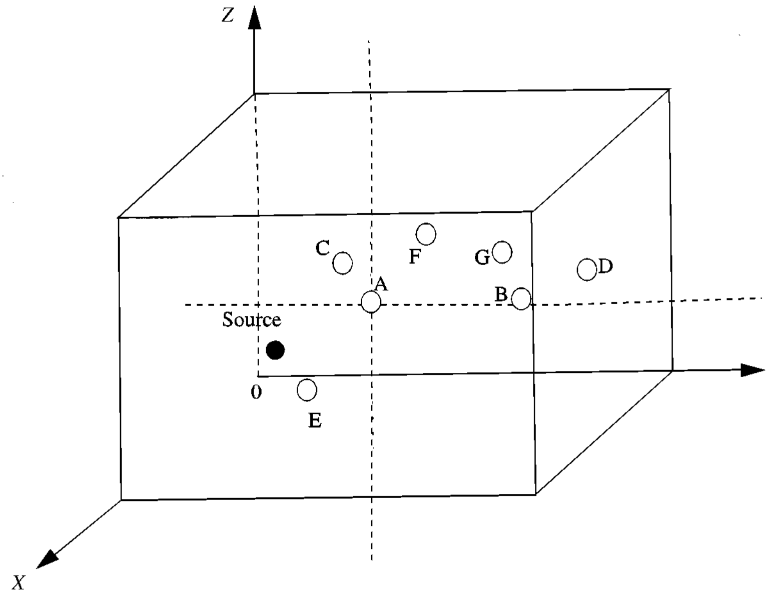

Figure 1. Positions of receivers in quasi-cubic space.

TABLE 2

Predicted and calculated SPL values in quasi-cubic space

Position	$SPL_{predicted}$ (dB)	$SPL_{calculated}$ (dB)	$SPL_{predicted} - SPL_{calculated}$ (dB)
A	78·64	78·82	− 0·18
B	78·50	78·75	− 0·25
C	79·62	79·03	0·59
D	78·70	78·76	− 0·06
E	78·95	78·85	0·10
F	78·38	78·76	− 0·38
G	78·53	78·72	− 0·19

and calculated values, are listed in Table 2. Table 3 shows the predicted values of the reverberation time and early decay time at each receiver position.

Table 2 shows that there is very little variation in the predicted sound pressure levels at different positions in the space, the biggest variation in predictions being 0·88 dB. This shows that NORMAL has predicted the characteristics of a diffuse sound field in this case. Furthermore, the values of SPL predicted by NORMAL agree with those calculated using Sabine theory, the average deviation between predicted and calculated SPLs being less than 0·5 dB.

From Table 3 it can be seen that the predicted EDT and RT values are very similar which again shows that NORMAL has modelled the sound field as diffuse,

TABLE 3

Predicted values of RT and EDT in quasi-cubic space

Position	$RT_{predicted}$ (s)	$EDT_{predicted}$ (s)
A	2·45	2·46
B	2·42	2·52
C	2·42	2·28
D	2·43	2·40
E	2·39	2·40
F	2·45	2·46
G	2·42	2·46

and the sound decays as linear in dB. Furthermore, the average value of both RT and EDT is 2·43 s, which is very close to the theoretical value given by the Sabine formula.

Thus, NORMAL has been shown to correctly model the characteristics of a diffuse sound field in this space.

3.2. THE LONG ENCLOSURE

The hypothetical long enclosure space was assumed to have dimensions $8 \times 100 \times 9$ m^3, with surface absorption coefficient 0·1. The source was 5 m from the left end in the middle of the cross-section, and the six receivers were 5–70 m from the source, in the centre of the cross-section along the length of the enclosure. The predictions were made with both ends open.

According to the Sabine formula, the reverberation time in this space is 3·25 s.

The calculated (using Sabine theory) and predicted sound pressure levels are listed in Table 4 and the predicted reverberation times and early decay times in Table 5. The tables show that the predicted sound field is non-uniform, the sound pressure level, reverberation time and early decay time changing with the source–receiver distance.

From Table 4 it can be seen that the predicted sound pressure level decreases rapidly with the increase in distance from the source, whereas those calculated by classical theory are constant beyond the direct sound field. This shows that NORMAL is correctly modelling the characteristics of a non-diffuse sound field in this space. The predicted reverberation times and early decay times increase gradually with increased distance from the source and differ significantly from the value calculated by the Sabine formula. There is also a difference between the predicted reverberation time and early decay time at each position, showing that NORMAL has correctly modelled the non-linear decay characteristic of a non-diffuse space.

These results agree with measurements of the sound field in long enclosures [1–3] which have shown that classical room acoustics is not applicable in these

TABLE 4

Calculated and predicted SPL in the long enclosure

Position	Distance (m)	$SPL_{calculated}$ (dB)	$SPL_{predicted}$ (dB)	$SPL_{predicted} - SPL_{calculated}$ (dB)
A	5	70·10	71·81	− 1·71
2	10	68·96	69·94	− 0·98
3	15	68·70	69·14	− 0·44
4	30	68·54	66·72	1·82
5	50	68·5	64·77	3·73
6	70	68·5	63·17	5·33

TABLE 5

Predicted RT and EDT in the long enclosure

Position	Distance (m)	$RT_{predicted}$ (ms)	$EDT_{predicted}$ (ms)
1	5	1·81	1·14
2	10	1·86	1·5
3	15	1·93	1·5
4	30	1·97	2·04
5	50	2·02	2·52
6	70	1·93	2·82

spaces, since the sound field is not uniform. Thus it has been shown that NORMAL is capable of modelling the variation in acoustic parameters which are typical of the non-diffuse sound field which occurs in long spaces.

4. REVERBERATION TIME TAIL COMPENSATION

Reverberation time tail compensation is the method used to compensate for the lack of later reflections recorded in ray tracing due to the speed and memory limitations of the computer. Approximations of this type have been used by Ondet and Barbry [12]. The feasibility of using this method in NORMAL for predictions in long enclosures has been examined by comparing predictions made with and without the compensation in both a diffuse space and a non-diffuse long enclosure.

4.1. CALCULATION OF RT TAIL COMPENSATION IN NORMAL

The reverberation time and early decay time are calculated from the reverberation curve in the NORMAL model using Schroeder's method [16]. The reverberation curve is calculated by reverse integration of the impulse response

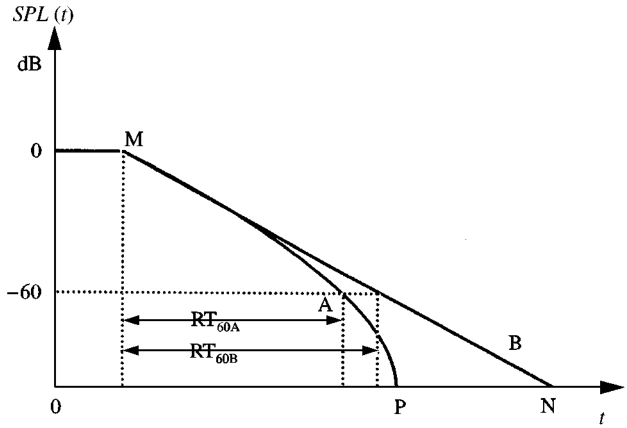

Figure 2. Actual and predicted reverberation curves.

from a certain time t to infinite time according to the following:

$$SPL(t) = 10\,\mathrm{lg} \int_t^{\infty} E(t)\,\mathrm{d}t \quad \mathrm{dB}, \tag{7}$$

where $E(t)$ is the impulse response of sound intensity or energy time decay.

From this curve, the reverberation time can be calculated as shown in Figure 2. The larger the space and the harder the surfaces, the longer the tracing time will be. In practice, due to the limitations of computer speed and memory, it is impossible to trace a ray until infinite time. Usually, the tracing of a ray will be terminated when the energy it carries has reduced to a negligible level, either after a predetermined number of reflections or as specified by the energy discontinuity percentage described above.

If the tracing time is not long enough, the reverberation curve will not be linear, and a certain amount of energy after the tracing time will be lost. For example, in Figure 2, the true reverberation decay is that shown by line B, from point M to point N; if the tracing time is reduced, the predicted decay curve will be as line A, from M to P. The reverberation time predicted from line A (RT_{60A}) will be less than that predicted by line B (RT_{60B}) as shown, and will thus be less than the actual reverberation time.

The method used to overcome this problem is to add the lost part of the energy to give a new reverberation curve. This is called the reverberation tail. This is based on the assumption that the sound field will be diffuse after the termination of the ray tracing, so the ray tracing method can then be replaced by the statistical method. The calculation method is as follows.

The energy in equation (7) is divided into two parts:

$$\int_t^{\infty} E(t)\,\mathrm{d}t = \int_t^{t_1} E(t)\,\mathrm{d}t + \int_{t_1}^{\infty} E(t)\,\mathrm{d}t, \tag{8}$$

where t_1 is the time at which the tracing of rays is terminated, $\int_t^{t_1} E(t)\,\mathrm{d}t$ is the energy obtained from the impulse response before time t_1, $\int_{t_1}^{\infty} E(t)\,\mathrm{d}t$ is the energy lost after the tracing of rays has been terminated.

Obviously, the lack of the second integral will cause systematic errors in the calculation of SPL values, and consequently generate an error in the reverberation time calculation.

Since it is assumed that the sound field is diffuse after time t_1, the second integral is a constant value and so can be represented by a constant Q_0. Thus, from equations (7) and (8)

$$SPL(t) = 10\lg\left(\int_t^{t_1} E(t)\,\mathrm{d}t + Q_0\right). \tag{9}$$

To find the value of the constant Q_0, the sound field is assumed to be quasi-diffuse, that is the $SPL(t)$ curve is assumed to decay linearly. Thus, the SPL curve can be represented as

$$SPL(t) = at + b. \tag{10}$$

The values of a and b in equation (10) can be determined by fitting the $SPL(t)$ curve to a linear equation using a non-linear least-squares method. The value of Q_0 can then be derived from the values of a and b using equations (9) and (10), and the reverberation time can thus be simultaneously calculated from the data fitting.

4.2. PREDICTIONS WITH AND WITHOUT REVERBERATION TAIL COMPENSATION

In order to study the effects of the reverberation tail compensation, and to examine its suitability in the case of long enclosures, predictions were made with and without the reverberation tail compensation in the same two hypothetical spaces as before. For this investigation, rather than using the energy discontinuity percentage to determine the ending of the tracing of a ray, the actual ray tracing time, that is, the length of time after which the tracing of each ray is terminated, was specified.

4.2.1. *Quasi-cubic space*

The reverberation time calculated by the Sabine formula is 2·38 s in the quasi-cubic space, as discussed previously. NORMAL was used to calculate the reverberation time in this space both with and without the reverberation tail compensation, and using two different ray tracing times.

Figures 3 and 4 show the predicted reverberation curves in the space with different ray tracing times. The tracing time in Figure 3 is 1 s, and in Figure 4 is 5 s.

Figure 3 shows the reverberation curves with and without the reverberation tail compensation. It can be seen that the two curves are significantly different in shape. With the compensation, the reverberation curve is a linear decay while without the compensation the reverberation curve decays more quickly and becomes non-linear towards the end of decay. The reverberation times with and without the reverberation time tail compensation are 2·4 and 2·08 s respectively. Thus, the prediction with the reverberation tail is the closer in value to that calculated using Sabine theory.

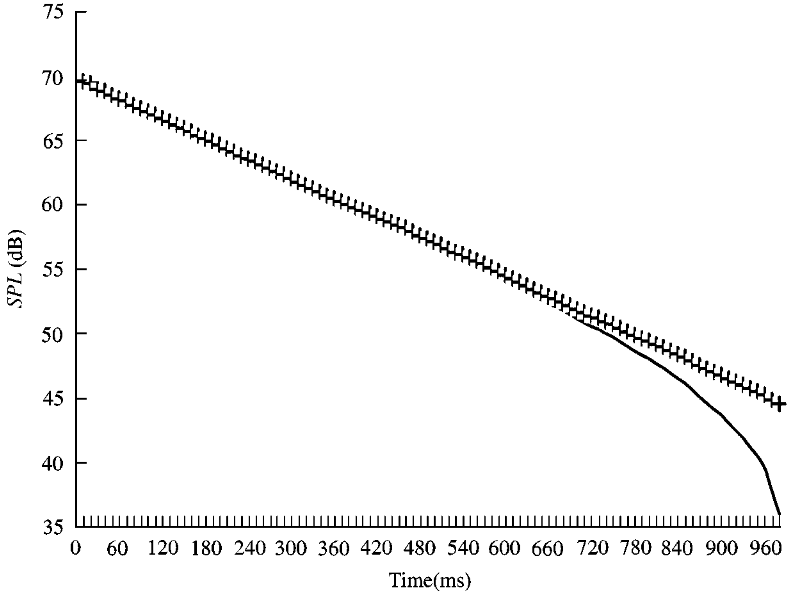

Figure 3. Predicted RT curves in quasi-cubic space, tracing time 1 s. —+— RT (tail), —— RT (no tail).

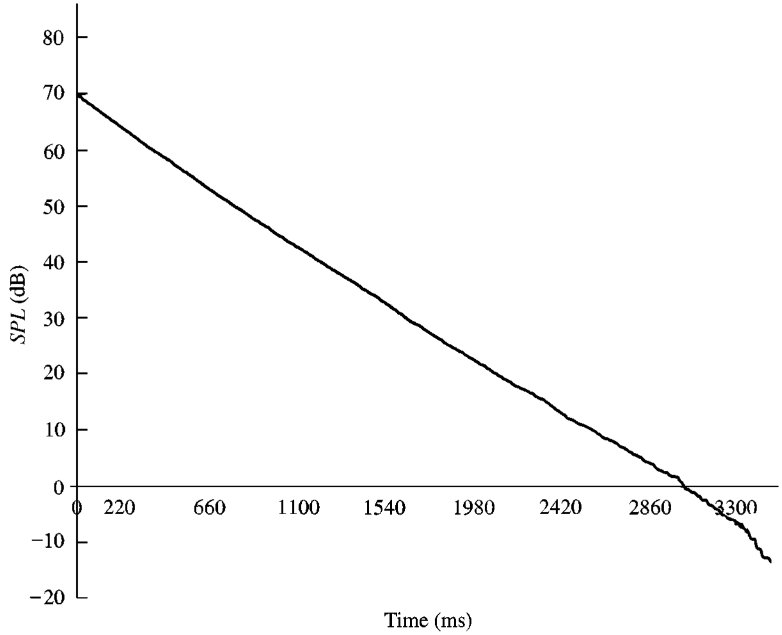

Figure 4. Predicted RT curve in quasi-cubic space, tracing time 5 s.

With the longer tracing time the predicted reverberation curves with and without the reverberation tail compensation are the same during the first 70 dB decay, as the energy carried by a ray has already reduced to vanishing point when the tracing is terminated. The curve predicted in both cases is linear as can be seen in Figure 4, and similar to the linear decay shown in Figure 3 with the compensation.

These calculations have shown that for a diffuse space the use of the reverberation tail compensation leads to accurate predictions of the reverberation time when the ray tracing time is comparatively short and late energy is lost. However, when a longer tracing time is used, the energy carried by a ray has already reduced to an insignificant level when tracing is terminated, so little energy is lost due to the termination of the rays. Hence, in this case, the curves are similar with and without the compensation and there is no need to compensate for lost energy. Thus, for a diffuse space the lost energy can be compensated for either by using the reverberation tail compensation, or by using a long ray tracing time.

The predicted curves shown in Figures 3 and 4 suggest that the compensation of energy is very important when the tracing time of rays is much shorter than the reverberation time. This is usually the case when the space is very big and the surfaces are acoustically hard. When the tracing time of the rays is larger than or comparable to the reverberation time, the difference between predictions with and without the compensation is negligible.

4.2.2. *Long enclosure*

The reverberation tail compensation is based on the assumption of a diffuse sound field after the termination of the ray tracing, which is not the case in a long enclosure. This method is therefore not directly applicable to this type of space and, as suggested in the case of the diffuse space above, a long ray tracing time might be more appropriate in order to allow for the later energy. To examine the feasibility of

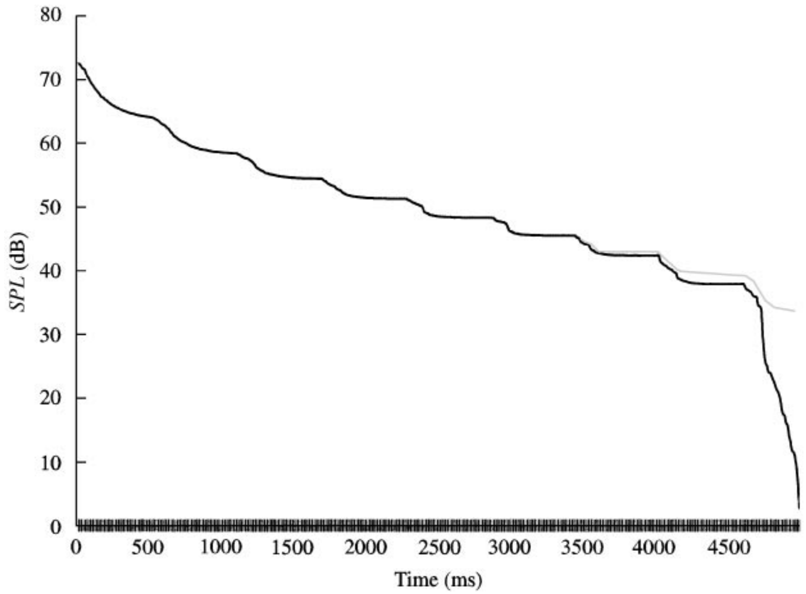

Figure 5. Predicted RT curves in long enclosure, tracing time 5 s. —— RT (tail), —— RT (no tail).

using the reverberation time tail compensation in a long enclosure, the reverberation curves were predicted by NORMAL using a tracing time of 5 s, with and without the compensation. The predicted decay curves for both cases are shown in Figure 5. The figure shows that the reverberation curve predicted from the impulse response using Schroeder's method is not a linear decay. The reverberation curves with and without the tail compensation are the same in the early part of the decay, as in the diffuse case with a short tracing time (see Figure 3). However, the curves deviate beyond a decay of approximately 35 dB. It can be seen that in this case, unlike the diffuse space, even with a relatively long tracing time, energy due to late reflections is lost due to the termination of the ray tracing. This is because the dimensions and volume of this space are large, so the sound paths are much longer than in the case of the quasi-cubic space. It appears that for a long enclosure the tracing time needs to be so long to take account of late reflections that it may be impractical in terms of computer time, in which case the reverberation tail compensation method could be used to increase the accuracy of results. However, the advent of faster and more powerful computers means that it may become feasible in future to use longer tracing times without correspondingly unreasonable run times.

5. CONCLUSIONS

The development of a ray tracing model, NORMAL, designed to model the sound field in convex spaces without curved surfaces has been described. The NORMAL model has been used to model sound propagation and reverberation and early decay times in two hypothetical spaces of differing shapes and sound field characteristics. It was shown that in a quasi-cubic space with a diffuse sound field, the sound pressure levels and reverberation times predicted by NORMAL were in agreement with those calculated using classical room acoustic theory. In a long enclosure, the values predicted by NORMAL varied throughout the space, showing that the model is also able to predict the characteristics of a sound field in a non-diffuse space.

The method of the reverberation tail compensation used to compensate for the lack of later reflection energy after the tracing time has been described. Use of this method in a diffuse space shows that it is useful when the tracing time is short compared to the reverberation time. To achieve accurate results in a long enclosure it is preferable to use a long tracing time, since the sound field is not diffuse. If the required tracing time is impractical owing to limitations of the computer, then the reverberation tail method could be used.

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